

Exercise 1. For each of the following, describe the steps that need to be taken to prove a statement of the given form:

(a) $(p \wedge q) \vee (r \wedge s)$

(b) $p \Rightarrow (q \wedge r)$

(c) $(p \wedge q) \Rightarrow r$

(d) $p \Rightarrow (q \Rightarrow r)$

(e) $(p \Rightarrow q) \Rightarrow r$

Exercise 2. Let n be a positive integer that divides 4. Prove that either n is even or n is a perfect square. (Hint: what are the possible values of n ? Which proof technique should you use?)

Exercise 3. Let x and y be real numbers. Prove that if x and $x + y$ are rational, then y is rational. (Recall that a number a is rational if it can be written as $\frac{p}{q}$, where p, q are integers and q is not zero)

Exercise 4. Let r be a rational number and a be irrational. Prove that $r + a$ is irrational (hint: do a proof by contradiction. The previous exercise may be useful).

Exercise 5. Let a and b be integers. Prove that if ab is even, then a is even or b is even.