log w/no base $\Rightarrow$ base 10 ; $\ln = \log_e$

$\log_b(x) \xleftrightarrow{\text{inverses}} b^x \Rightarrow [\log_b(x) = y \Longleftrightarrow b^y = x]$

Review for Math 121 Final:

Question 1:

Simplify:

a) log(1,000,000) $= 6 \Longleftarrow 10^6 = 1{,}000{,}000$

b) $\log(\frac{1}{10}) = -1 \Longleftarrow 10^{-1} = \frac{1}{10}$

c) $log_2 16 = 4 \Longleftarrow 2^4 = 16$

d) $log_7 49 = 2 \Longleftarrow 7^2 = 49$

e) $log_2(\frac{1}{16}) = -4 \Longleftarrow 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

$\log_b(x) + \log_b(y) = \log_b(xy)$ | $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$

Question 1, cont'

f) $log_9 1$ $= \boxed{0}$ $\Longleftarrow$ $9^0 = 1$

g) $log_2 24 \ - \ log_2 3$

$= \log_2\left(\frac{24}{3}\right) = \log_2(8) = \boxed{3}$ $\Longleftarrow$ $2^3 = 8$

h) $ln(e^3)$ $= \boxed{3}$

i) $log(4) \ + \ log(25)$ $= \log(4 \cdot 25) = \log(100) = \boxed{2}$

$10^2 = 100$

Question 2:

Expand the following expressions

$\sqrt[n]{x} = x^{1/n}$. $n\log_b(x) = \log_b(x^n)$

a) $ln(m^2 n^5 \sqrt{p})$

$= \ln(m^2) + \ln(n^5) + \ln(\sqrt{p}) = \boxed{2\ln(m) + 5\ln(n) + \frac{1}{2}\ln(p)}$

b) $log(\frac{7x^4}{\sqrt[3]{y}})$

$= \log(7x^4) - \log(\sqrt[3]{y}) = \log(7) + \log(x^4) - \log(y^{1/3})$

$= \boxed{\log(7) + 4\log(x) - \frac{1}{3}\log(y)}$

Question 3:

Solve the following logarithmic equations for x:

a) $log(x) + log(x + 21) = 2$

$\Downarrow$

$$\log(x(x+21)) = 2 \Rightarrow \log(x^2+21x) = 2$$

$$\Rightarrow 10^{\log(x^2+21x)} = 10^2 \Rightarrow x^2+21x = 100 \Rightarrow$$

$$x^2+21x-100=0 \Rightarrow (x+25)(x-4)=0 \Rightarrow \boxed{x=4} \text{ or } \cancel{x=-25}$$

b) $log_x 9 = 2$

$\Downarrow$

$$x^{\log_x 9} = x^2 \Rightarrow 9 = x^2 \Rightarrow \boxed{x=3} \text{ or } \cancel{x=-3}$$

domain of log is $(0, \infty)$

c) $log_3 x = 4$

$\Downarrow$

$$3^{\log_3 x} = 3^4 \Rightarrow \boxed{x = 81}$$

Question 4

For the polynomial below, state the zeros and their multiplicities. Find the y-intercept, end behavior and sketch a graph.

$$y = \underbrace{(x^2 - 9)}_{\text{factor}}(x + 1)(x - 2)^2 = (x+3)(x-3)(x+1)(x-2)^2$$

— so the zeros are : $-3,\ 3,\ -1,\ 2$

w multiplicities : $1 \quad 1 \quad 1 \quad 2$

— y-int $= (0^2-9)(0+1)(0-2)^2$

$= (-9)(1)(-2)^2 = -9 \cdot 4 = -36$

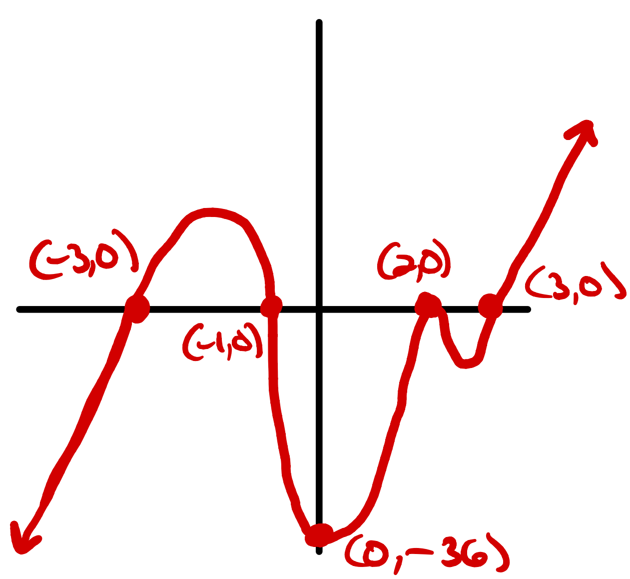
— end behavior: degree $= 1+1+1+2 = 5$ odd

leading coefficient $= 1 > 0$

so

as $x \to \infty,\ y \to \infty$

as $x \to -\infty,\ y \to -\infty$



Question 5:

For the rational functions below:

State any x-intercepts, y-intercepts, vertical asymptotes, horizontal asymptotes, “holes” (in point form). And then sketch a graph for the given function.

a) $y = \frac{x^2+5x+6}{x^2-9} \xrightarrow{\text{factor}} \frac{(x+3)(x+2)}{(x+3)(x-3)}$

$\left\{ \begin{array}{l} \text{zeros of top : } -3, -2 \\ \text{zeros of bottom : } -3, 3 \end{array} \right\}$

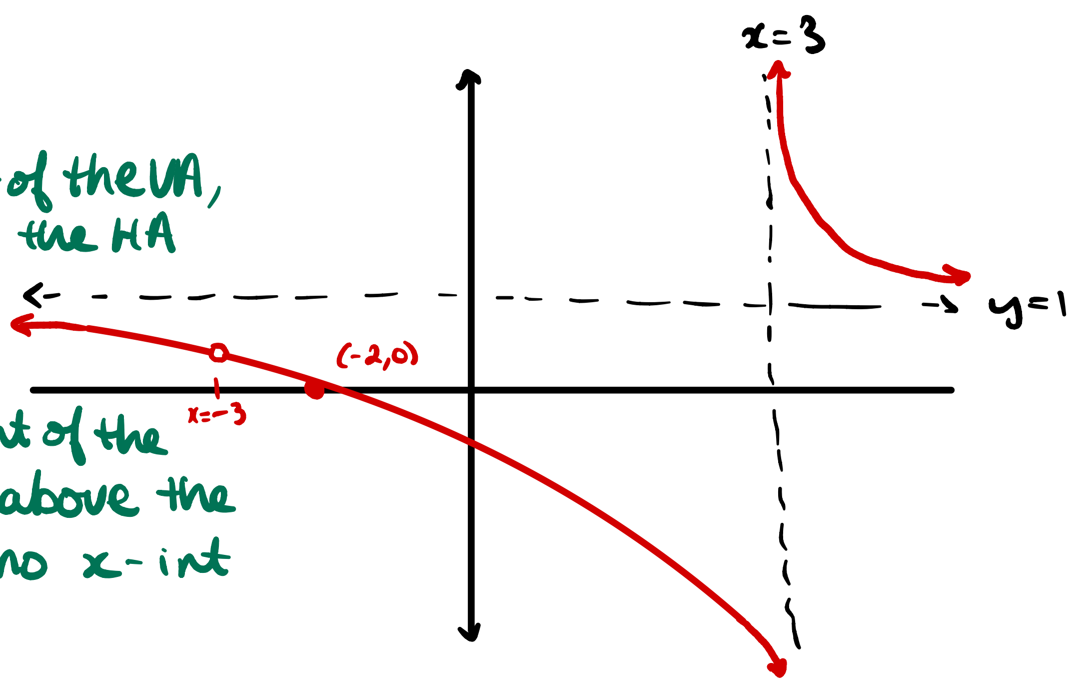
- Hole(s): $x=-3$ (zero of both)
- V.A.(s): $x=3$ (zero of bottom <u>only</u>)
- x-int(s): $x=-2$ (zero of top <u>only</u>)

$$\text{y-int} = \frac{0^2+5\cdot 0+6}{0^2-9} = \frac{6}{-9} = -\frac{2}{3}$$

let $\frac{f(x)}{g(x)}$ be a rational fn

- degree of f = degree of g
  $\Rightarrow$ HA @ $y = \frac{\text{leading coeff. of top}}{\text{leading coeff. of bottom}}$
- degree f < degree g
  $\Rightarrow$ HA @ $y=0$
- deg f > deg g
  $\Rightarrow$ no HA

HA: $y=1$




We know that left of the VA, the graph is below the HA because there is an x-int.

We know that right of the VA, the graph is above the HA b/c there is no x-int

b) $y = \frac{x^2+4}{(x-4)^2}$ $\rightsquigarrow$ can't factor ( top has no roots in $\mathbb{R}$)

$\left\{ \begin{array}{l} \text{zeros of top: none} \\ \text{zeros of bottom: } 4 \end{array} \right\} \Rightarrow$

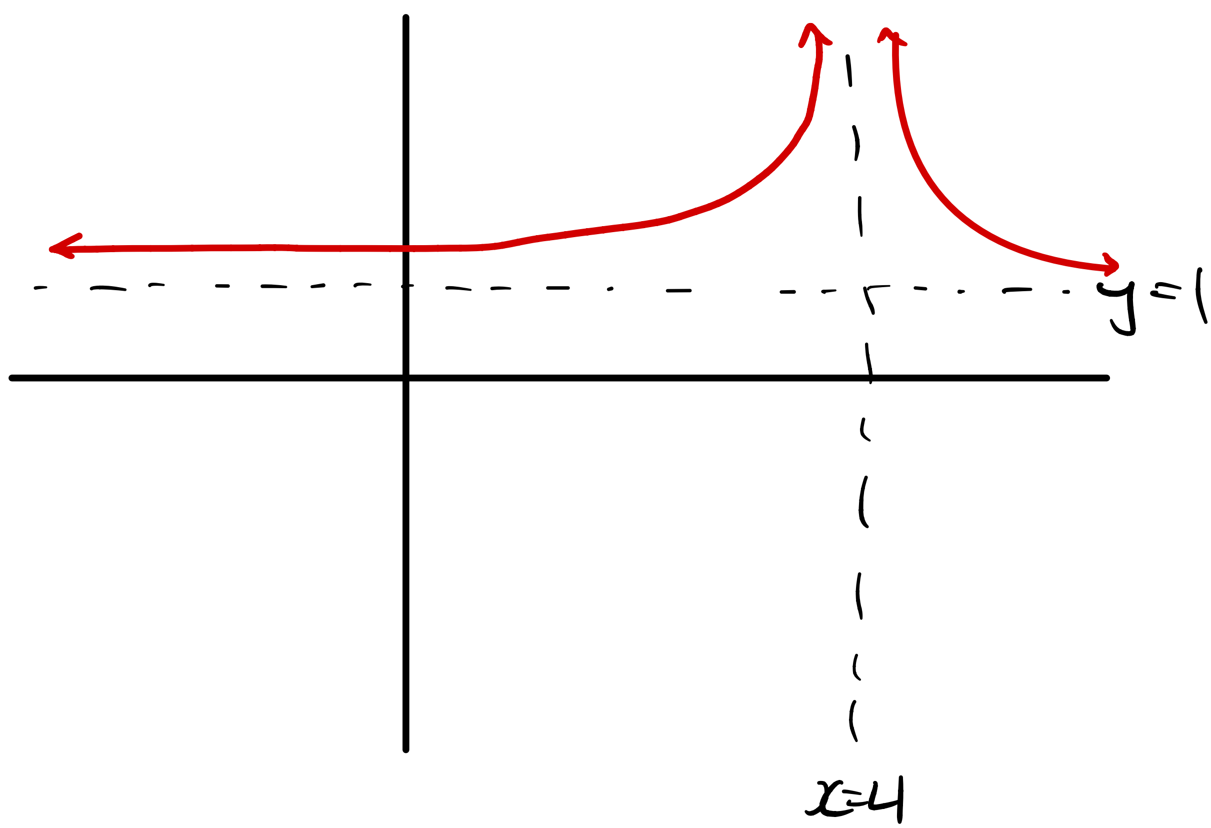
Hole(s): none

VA(s): $x=4$

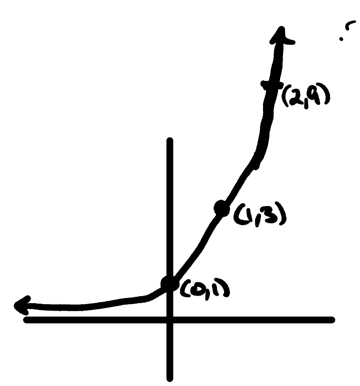
x-int(s): none

$y\text{-int} = \frac{0^2+4}{(0-4)^2} = \frac{4}{16} = \frac{1}{4}$

HA @ $y=1$



Base fn: $3^x$ ⇝




Question 6:

Consider the function: $f(x) = 3^{x+4} - 2$

a) Give the domain of f $\mathbb{R}$

b) State the y-intercept (if any) in point form.

$$3^{0+4} - 2 = 3^4 - 2 = 81 - 2 = 79 \quad \boxed{(0, 79)}$$

c) state the horizontal asymptote (if any)

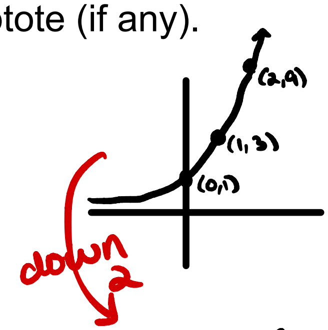
$3^{x+4} \underbrace{-2}_{\text{shift down 2}}$. Base function has HA @ y=0 ⇝ $\boxed{\text{HA @ } y = -2}$

d) State the vertical asymptote (if any).

none

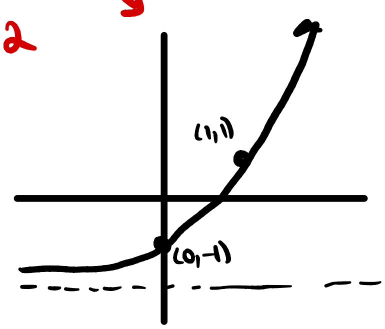
e) Give the range of f.

$(-2, \infty)$

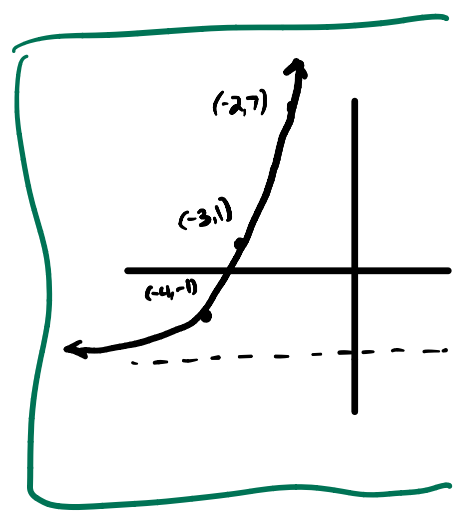



f) Sketch the graph of f.

$3^{\underbrace{x+4}_{\text{shift left 4}}} \overbrace{-2}^{\text{down 2}}$




left 4 →



Question 7:
Solve the following equations:

a) $2^{x+3} = \frac{1}{4}$

b/c $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\Downarrow$

$\log_2(2^{x+3}) = \log_2\left(\frac{1}{4}\right) = -2 \Rightarrow x+3 = -2 \Rightarrow \boxed{x = -5}$

b) $3^{x+7} = 81$

b/c $3^4 = (3^2)^2 = 9^2 = 81$

$\Downarrow$

$\log_3(3^{x+7}) = \log_3(81) = 4 \Rightarrow x+7 = 4 \Rightarrow \boxed{x = -3}$

c) $5^{x+2} = 3$

$\Downarrow$

$\log_5(5^{x+2}) = \log_5(3) \Rightarrow x+2 = \log_5(3) \Rightarrow \boxed{x = \log_5(3) - 2}$

d) $2 \cdot 3^{x+1} = 18$

$\Downarrow$

$3^{x+1} = \frac{18}{2} = 9$

$\Downarrow$

$\log_3(3^{x+1}) = \log_3(9)$

$\Downarrow$

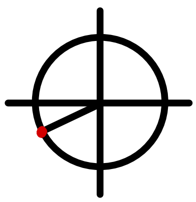
$x+1 = 2$

$\Downarrow$

$\boxed{x = 1}$

Question 8:
Evaluate:

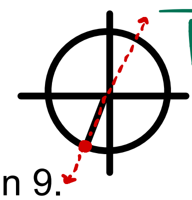
$sin(\frac{7\pi}{6})$



$\boxed{-\frac{1}{2}}$

$cos(\frac{7\pi}{6})$ $\boxed{-\frac{\sqrt{3}}{2}}$

tan($\frac{4\pi}{3}$)



$\boxed{\sqrt{3}}$

Question 9:
What is the domain and range for sin(x)?

$\mathbb{R}$ $\quad [-1,1]$

What is the domain and range for tan(x)? $\to \mathbb{R}$

$$\{x \in \mathbb{R} \mid x \neq \tfrac{\pi}{2} + \pi k \text{ where } k \text{ is an integer}\}$$

$$= \mathbb{R} \setminus \{\tfrac{\pi}{2} + \pi k \mid k \in \mathbb{Z}\}$$

= ... multiple equiv. ways to write it.

just remember $\tan(x) = \frac{\sin x}{\cos x}$. so

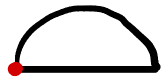
$\tan x$ undefined when $\cos x = 0$.

$\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$

So domain(tan) = all real #s except $\frac{\pi}{2} + \pi k$, where $k \in \mathbb{Z}$

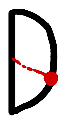
Question 10

a) $cos^{-1}(1)$



$\boxed{\pi}$

b) $sin^{-1}(-\frac{1}{2})$



$\boxed{-\frac{\pi}{6}}$

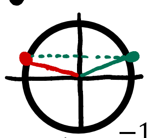
c) $cos^{-1}(cos(2.5))$

$0 \le 2.5 \le \pi = 3.1415\ldots$

so $= \boxed{2.5}$ since $2.5 \in \text{range}(\cos^{-1})$

d) $sin^{-1}(sin(\frac{5\pi}{6}))$

$\frac{5\pi}{6} \notin \text{range}(\sin^{-1})$ since $\frac{5\pi}{6} > \frac{\pi}{2}$

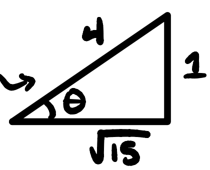


$\boxed{\frac{\pi}{6}}$

e) $tan(sin^{-1}(1/4))$

let $\sin^{-1}(\frac{1}{4}) = \theta$

then $\sin\theta = \frac{1}{4}$ $\rightsquigarrow$




$4^2 = 1^2 + b^2 \Rightarrow \sqrt{15} = b$

$\tan(\sin^{-1}(\frac{1}{4}))$

$\shortparallel$

$\tan\theta$

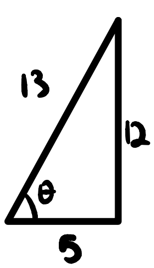
$\shortparallel$

$\boxed{\frac{1}{\sqrt{15}}}$

f) $sin(cos^{-1}(5/13))$

let $\cos^{-1}(5/13) = \theta$

then $\cos\theta = 5/13$




$13^2 = 5^2 + a^2$

$\Downarrow$

$169 - 25 = a^2$

$\Downarrow$

$144 = a^2$

$\Downarrow$

$a = 12$

$\sin(\cos^{-1}(\frac{5}{13}))$

$\shortparallel$

$\sin\theta$

$\shortparallel$

$\boxed{\frac{12}{13}}$

Question 11

What is the domain and range for $sin^{-1}(x)$?

$[-1,1]$ $\quad$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$

What is the domain and range for $cos^{-1}(x)$?

$[-1,1]$ $\quad$ $[0,\pi]$

Question 12

$\underbrace{sin(\theta) = 2/3}_{2/3>0}$ and $\underbrace{cos(\theta) < 0}$ then find the value of the other five trig functions

QI or QII $\quad$ QII or QIII

QII $\Rightarrow$ sin, csc $>0$
cos, sec, tan, cot $<0$

$sin(\theta) = 2/3$

cos(θ) = $-\frac{\sqrt{5}}{3}$

tan(θ) = $-2/\sqrt{5}$

csc(θ) = $3/2$

sec(θ) = $-3/\sqrt{5}$

cot(θ) = $-\sqrt{5}/2$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow (\frac{2}{3})^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{4}{9} = \frac{5}{9}$

$\Rightarrow \cos\theta = -\sqrt{5}/3$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/3}{-\sqrt{5}/3} = -\frac{2}{\sqrt{5}}$

$\csc\theta = \frac{1}{\sin\theta}$

$A\sin(\omega(x-b)) + m$

Question 13
For each function give:
a) amplitude
b) midline
c) max value
d) min value
e) period

i) f(x) = $7\sin(6\pi x) - 1$

a) 7

b) $y=-1$

c) $-1+7=6$

d) $-1-7=-8$

e) $\frac{2\pi}{\omega} = \frac{2\pi}{6\pi} = \frac{1}{3}$

ii) g(x)= 3cos(2x) + 4

a) 3

b) $y=4$

c) $4+3=7$

d) $4-3=1$

e) $\frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Question 14

Solve 2sin(3x) + 1 = 0

a) Give the general solution

b) Give the solutions for x which which lie between [0,2π)

$$2\sin(3x)+1=0 \Rightarrow 2\sin(3x)=-1 \Rightarrow \sin(3x)=-\frac{1}{2}$$

So

$$3x = \frac{7\pi}{6} + 2\pi k$$

$$3x = \frac{11\pi}{6} + 2\pi k$$

Then

$$x = \frac{1}{3}\left(\frac{7\pi}{6} + 2\pi k\right)$$

$$x = \frac{1}{3}\left(\frac{11\pi}{6} + 2\pi k\right)$$

a)

$$\Rightarrow \boxed{\begin{aligned} x &= \frac{7\pi}{18} + \frac{2\pi k}{3} \\ x &= \frac{11\pi}{18} + \frac{2\pi k}{3} \end{aligned}}$$

To find solns on [0, 2π) we need to figure out what values we can plug in for k. It helps to rewrite:

$$x = \frac{7\pi}{18} + \frac{2\pi k}{3}\cdot\frac{6}{6} = \frac{7\pi + 12\pi k}{18}, \quad x = \frac{11\pi}{18} + \frac{2\pi k}{3}\cdot\frac{6}{6} = \frac{11\pi + 12\pi k}{18}$$

and note $2\pi = \frac{36\pi}{18}$.

Then

$\cancel{k=-1}$ $\downarrow$ $\underbrace{-\frac{5\pi}{18}, -\frac{\pi k}{18}}_{<0}$

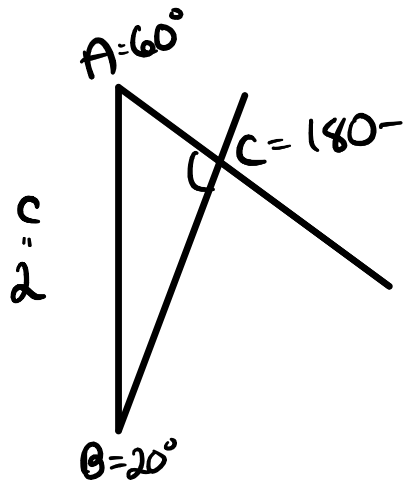
k = 0, k = 1, k = 2, $\cancel{k=3}$ too big

$$\boxed{\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}}$$

Question 15

Solve the following triangle:

A = 60 degrees, B = 20 degrees, c = 2. Write down the exact values for C, a and b.




$C = 180 - 60 - 20 = 100°$ $\rightsquigarrow$ $C = 100°$

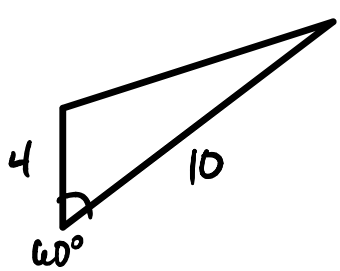
$$\frac{c}{\sin C} = \frac{2}{\sin 100}$$

$$\frac{2}{\sin 100} = \frac{a}{\sin 60} \Rightarrow a = \frac{2\sin 60}{\sin 100}$$

$$\frac{2}{\sin 100} = \frac{b}{\sin 20} \Rightarrow b = \frac{2\sin 20}{\sin 100}$$

Question 16

Given a triangle with A = 60 degrees, b = 10, c= 4. Find the exact value of a



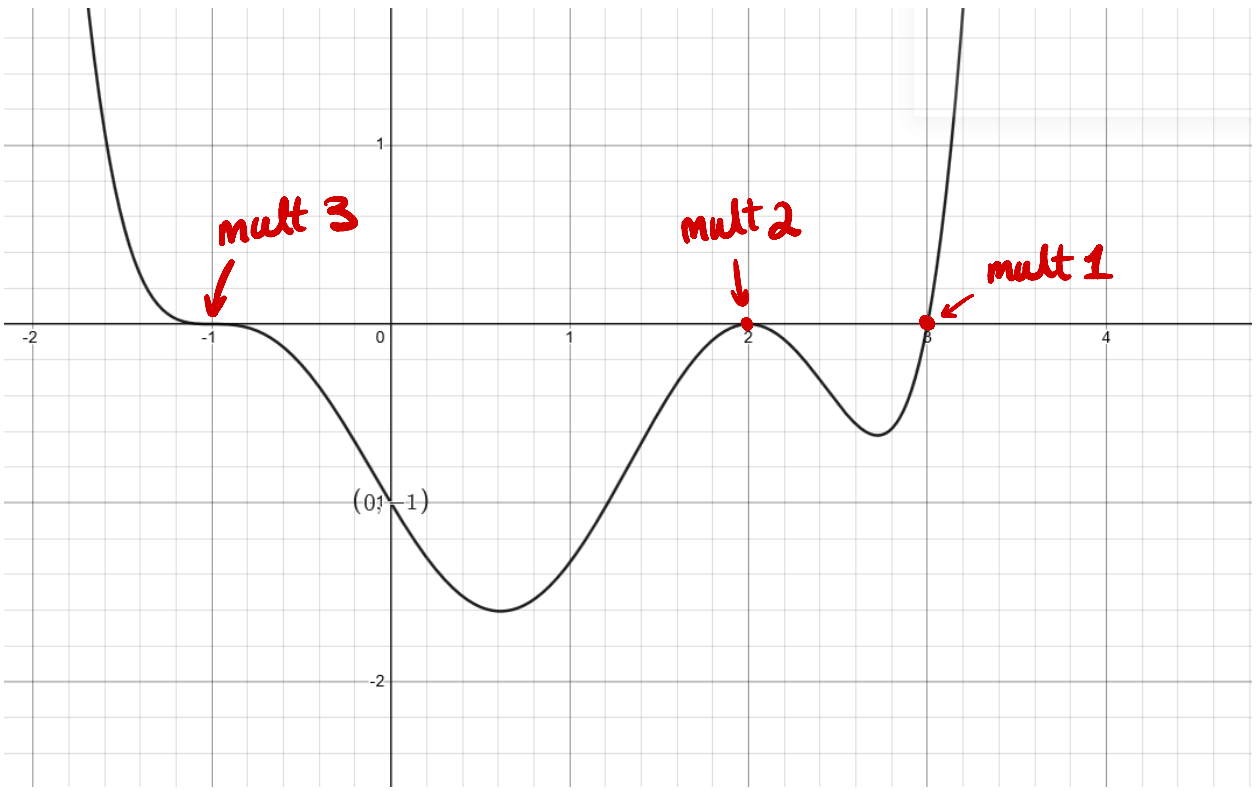

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 16 + 100 - 2(4)(10)\cos 60$$
$$= 116 - 80 \cdot \tfrac{1}{2}$$
$$= 116 - 40$$
$$= 74$$

$\Rightarrow a^2 = 74$

$\Downarrow$

$$\boxed{a = \sqrt{74}}$$

Question 17




For the above function: state the zeroes and their multiplicities. State the y-intercept. And then give a possible formula for the function.

| zeros | -1 | 2 | 3 |
|---|---|---|---|
| mults | 3 | 2 | 1 |

so the factors of the polynomial are

$(x+1)^3, \ (x-2)^2, \ (x-3)$

let $f(x) = A(x+1)^3(x-2)^2(x-3)$

we know

$-1 = f(0) = A \cdot (1)^3 \cdot (-2)^2 \cdot (-3) = -12A$

$\Rightarrow A = \frac{1}{12}$

$f(x) = \frac{1}{12}(x+3)^3(x-2)^2(x-3)$