olynomials	
<u></u>	
$f(x) = a_n x^n + a_n$	-1 X ⁿ⁻¹ + + a, x + a, where a; eR, an ≠0 is a polynomial of degree n, with leading coefficient an and y-intercept (0,f(0))
less formally:	given a polynomial, its degree is the highest power of x that appears and its leading coefficient is the coefficient of the highest power of
examples. X 7x	$x^3 - 2x + 1$ has degree 5, and leading coefficient 1 $x^2 - 2x^4 - x^3$ has degree 4, and leading coefficient -2
-(x) = A(x - c,)" (x	(-c2) ^{m2} (x-cr) ^{m2} is the factored form of a polynomial with degree m1+m2++m
	and leading coefficient A
The factored Each C; is a	form allows us to find the zeros/z-intercepts of fix) and their multiplicities. zero, with multiplicity m;
examplesx(x-	$2^{3}(x+7)^{3}$ is a polynomial of degree $1+3+2=6$, with leading coefficient -1.
	the elios are: x=0, or (0,0), with multiplicity 1
	X=2, or (2,0), with MULTIPLICITY 3
	x=-7, or (7,0), with multiplicity 2
0/	$(1, 1)^2$
9(X+)) 3x-21 is a polynomial of degree 2+1=2, with waainy coefficient 7.
when tites	you have a time the zeros oul: z=-s, or (syo), with multiplicity z this set it equal
to zer	e and selve for x L= 5, or croir, while many mentions at
5x-2	=0 → λ = 5 is the zero
That does de 6 a polynomial	gree, leading coefficient, zeros/multiplicities tell us about the graph
nd behavior:	The ends of an even degree polynomial go in the same direction,
	if the leading coefficient is positive: As $x \rightarrow \infty$, $y \rightarrow \infty$ and as $x \rightarrow -\infty$ $y \rightarrow \infty$.
	IT THE MUNICIPY WEBBILIER IS MAGALINE HS X-300, 4-3-00 and as X-3-00 4-3-00 45.
	The ends of an ald dearee polynomial in the martine structions
	The ends of an odd degree polynomial go in opposite directions,
	The ends of an odd degree polynomial go in opposite directions, if the leading coefficient is positive: As $x \rightarrow \infty$, $y \rightarrow \infty$ and as $x \rightarrow -\infty$ $y \rightarrow -\infty$
	The ends of an odd degree polynomial go in opposite directions, if the leading coefficient is positive: $As x \rightarrow \infty, y \rightarrow \infty$ and $as x \rightarrow -\infty y \rightarrow -\infty$ if the leading coefficient is $As x \rightarrow \infty, y \rightarrow -\infty$ and $as x \rightarrow -\infty y \rightarrow -\infty$
Behavior at <i>x-in</i>	The ends of an add degree polynomial go in apposite directions, if the leading coefficient is positive: As x→∞, y→∞ and as x→-∞ y→-∞ if the leading coefficient is As x→∞, y→∞ and as x→-∞ y→∞ tercepts: A zero with multiplicity 1 passes straight through the x-axis ←
Behavior at z-in	 The ends of an add degree polynomial go in apposite directions, if the leading coefficient is positive: As x→∞, y→∞ and as x→-∞ y→-∞ if the leading coefficient is As x→∞, y→∞ and as x→-∞ y→∞ tercepts: A zero with multiplicity 1 passes straight through the x-axis A zero with even multiplicity bounces off of the x-axis
Behavior at z-in	The ends of an add degree polynomial go in apposite directions, if the leading coefficient is positive: As x→∞, y→∞ and as x→-∞ y→-∞ if the leading coefficient is As x→∞, y→∞ and as x→-∞ y→∞ tercepts: A zero with multiplicity 1 passes straight through the x-axis ← A zero with even multiplicity bounces off of the x-axis ← 1/ox
Behavior at z-in	 The ends of an odd degree polynomial go in apposite directions, if the leading coefficient is positive: As x→∞, y→∞ and as x→-∞ y→-∞ if the leading coefficient is As x→∞, y→∞ and as x→-∞ y→∞ tercepts: A zero with multiplicity 1 passes straight through the x-axis A zero with even multiplicity bounces off of the x-axis A zero with add multiplicity passes through the x-axis A zero with add multiplicity the diates the line

examples.	$\operatorname{graph} f(\mathbf{x}) = -\mathbf{x}^3 - 2\mathbf{x}^4 + 3\mathbf{x}$
	First, we factor $f(x)$. We pull out the common term (-x) to get $f(x) = -x(x^2+2x-3)$
	Then, since (-2)-(-1)=2 and (-1)-(-3)=-3, we factor the quadradic, getting f(x)= -x(x-2)(x-3)
	we can now see that the zeros are at (0,0), (2,0), and (3,0) and each has multiplicity 1
	Next use note that f has degree 3 (add) and hading conflictent -1. "the end behavior is
	The grapher, we griss provide at the
	then since as $\chi \to -\infty$, $\gamma \to \infty$, we draw a line going is up and to the reft statting at (1).
	and since as $x \rightarrow \infty$, $y \rightarrow \infty$, we draw a line going down and to the right starting at (3,0)
	Then since the multiplicity of each line is 1, at each zero we go straight through the
	χ -axis. It is usually best to work left \rightarrow right
	(63)
	Write a possible function for the graph
	of the degree 5 polynomial
	First, we see that the zeros are at (20) (4.0)
	$x = -2$, with multiplicity 2, giving us the term $(x+2)^{2}$ and
	$x = 4$, with multiplicity 3, giving us the term $(x-4)^3$
	So we know $f(x) = A(x+2)^{3}(x-4)^{3}$
	we use the u-intercept to find the leading coefficient A.
	The u-int. is (0.2) so we know $f(0)=2$. From mir formula, we know $f(0)=A(0+2)^3(0-4)^3$
	$f(x) = 0 = -560$ $e^{-1} = -\frac{1}{2}$
	$f(x) = -\frac{1}{(2^{6})^{2}} (x+2)^{2} (x-4)^{3}$
D Lucal	Curreliand
Kationa	t functions
	P
y p(x), q	(x) are polynomials, $f(x) = \overline{g(x)}$ is a rational function.
· A hole / re	emovable point h is a zero of both p(x) and g(x), he, a zero of both the numerator and denominator.
- An z-inter	capt of f(x) is a zero of p(x) that is not a hole
· A vertical	asymptote of fcx) is a zero of qcx) that is not a hole
examples.	$f(x) = \frac{3(x-2)(x+1)(x+3)}{x+3}$ has a hole at $x = -3$, x-intercepts (2,0) and (-1,0), and a V.A. at $x=0$
-,,	x (x+3)
	$f(x) = \frac{-x^2+4}{(x+2)(x+2)}$ has a hole at $x=-2$, an x-interval (9.0) and no VA.
L	
0	0
Horizontal	nsymptons:

Let n be the degree of p(x) and let m be the degree of q(x). Then there are 3 cases that $f(x) = \frac{p(x)}{q(x)}$ could fall into:

1. m=n: but a be the leading coefficient of p(x) and but b be the leading coefficient of q(x). Then f(x) has a H.A. at $y=\frac{1}{2}$ (This is because the numerator and denominator grow at the same rate

instra	f(I) Nus															
when	n=m+1,	f(x) has an	oblique	asymptote	. We	can	find the	equal	ion fo	r the	0.A.	throug	h po	Ny no M	vial	
										lon	g divi	sion	9(x)	Pro		
		3 x 2 + 2x - 4														
example	5. .	x ⁵ -3	- ha	s a HA	l al	y=0 .										
		×2-9														
		x2+x+5	has	a H.A.	. at .y=	1										
		~3. 272.1.														
		x2-4	has 1	no H.A.,	but do	es have	e an O	A., the	line	x+2.						
				x+2	→0.A. -											
			X*-	-4 1x3+2x2-1												
				213-	<u>(x</u>) 											
				-/2-2								· ·				
				(<u></u>	- 3)											
			• •		- 81 - 42-7	→disco	ud	• •			• •	•		•		
••••	• • •		• •		- 31 - 4z-7)-	> disco	urd	0 0	•		• •	•	• •	•	• •	0
· · ·		· · · ·	• •		- 8) - 42-7)-	>disco	ud	· ·	•	· ·	• •	•	· ·	•	• •	•
Using	X-inter	upts, asympt	otes, an	d test p	- 81 -4z-7 0ints,	-disco we (ud can gra	ph a	ratio	nal	funct	ion, o	rdu	ıtırm	ine .	the
Using formula	x-intere-of a roo	upts, asympt lional function	otes, an from a	d test p graph.	- <u>-</u> 31 (4z-7) Oints,	→disco we (ud can gra	ph a	ratio	nal	funct	ion, o	r du	ıterm	ine -	the
Using formula	x -intere- of a roo	upts, asympt tional function	otes, an from a	d test p graph	- <u>-31</u> -4z-7 0ints,	-disco we (ud can gra	pha.	ratio	nal	funct	ion, o	r du	ıtırm	ine	the
Using Jormula Xamples.	2-intera -of a roo graph	upts, asympt tional function $f(x) = \frac{x^3}{x^3}$	otes, an from a ² -5x-6	d test p graph.	- <u>-</u> 31 (4z-7)	→disco	con gra	ph a	ratio	nol	funct	ion, o	r da	ıtırm	ine	the the
Using formula :xamples.	x-intera -of a roo graph	upts, asympt tional function $f(x) = \frac{x^3}{x^5}$	otes, an from a -5x-6 '-4z	d test p graph.	6 ints,	→disco	ud can gra	pha a	ratio	nal 2)(x	funct - 3)(1	ion, o (+1)	r da	iterm	ine -	the
Using Jormula Xamples.	x-intera -of a roo graph We	epts, asympt tional function $f(x) = \frac{x^3}{x^5}$ first factor	otes, an from a '-5x-6 '-4x the num	d test p graph.	oints,	→disco we (ud can gra clor:	рћа 1 f(x)=	ratio	nal 2)(x (x+2)	funct - 3)(1 (x-2)	(r)	r da	iterm	ine -	the
Using Jormula :xamples.	z-intera -of a roo graph We Hoks	epts, asympt tional function $f(x) = \frac{x^3}{x^5}$ first factor (x+2) is a	otes, an from a ² -5x-6 ² -4z the num factor of	d test p graph. wrator av of both H	oints, ad du	-disco we (nomina merate	und can gra utor: sr 4 dur	ph a f(x)=	ratio	nal 2)(x (x+2) 50 f	funct - 3)(1 (x-2) here	(+1) (5 a	r du	iterm	ine	the
Using Jormula Xamples.	z-intera of a roo graph We Hoks z-ints	epts, asympt tional function $f(x) = \frac{x^3}{x^5}$ first factor (x+2) is a $(x-3)_{x}(x+1)$ and	otes, an from a -5x-6 -4x the num factor factor	d test p graph. rerator ar of both the	oints, vd. du ve nu	-disco we (nomina marata but n	und can gra ector: sr 4 der st the c	ph a f(x)= lomina	ratio	nal 2)(x (x+2) so t ther	-3)(7 (2-2) here e are	(+1) (+1)	r de hole	eterm at cepts	x=-2	the
Using Jormula Xamples.	z-intera of a roo graph We Hoks z-ints V:A.s	epts, asympt tional function $f(x) = \frac{x^3}{x^3}$ first factor (x+2) is a (x-3),(x+1) or (x) and (x)	otes, an from a -5x-6 -4x the num factor factors of x-2) are	d test p graph. urator ar of both H the nume lockors c	vite-7	we a	ud can gra stor: sr 4 der st the c m. but	ph a f(x)= comina denom not th	(x- x) tor, , so	nal (2)(x (x+2) 50 t ther m. sc	-3)(7 (2-2) here e are there	(+1) (s.a. (x-1)	r de hole inter	eterm at cepts s at	x= -2 (3,0 x=0	the 2 0), (
Using formula :xamples.	z-intera of a roo graph We thoks z-ints V:A.s Sinra	epts, asympt tional function $f(x) = \frac{x^3}{x^3}$ first factor (x+2) is a (x-3),(x+1) and (x) and (f is undefine	otes, an from a 2-5x-6 2-4x the num factor of factors of x-2) are d for x	d test p graph. graph. both the fue nume factors c =0 (V.R)	vite-7	we a nomina merata but n oleno	etor: stor: st the c m. but	ph a F(x) = comination heno	ratio (x- x) tor, , so t nu	nal 2)(x (x+2) 50 t ther m.;sc	funct -3)(7 (x-2) here e are , there	(+1) (+1) (5 a x-1 (are	r de hole v.A	at cepts s at	x= -2 (3,0 x=0	the 2 0), (and
Using formula :xamples.	z-intera of a roo graph We T Holes z-ints V:A.s Since Since	epts, asympt tional function $f(x) = \frac{x^3}{x^3}$ first factor (x+2) is a (x-3),(x+1) and (x) and (f is undefine the denomin	otes, an from a 2-5x-6 -4x the num factors of x-2) are d for x	d test p graph. graph. both the the nume factors c =0 (V.A) 3 degree 3	vite-7) oints, oints, vite nu urator, rf the there	we a nomina marate but n oleno is r	ector: ector: extor: ext the c m. but the y-in numero	ph a f(x)= comination how the tercept tor how	(<u>x-</u> x) tor, , so t nu s du	nal 2)(x (x+2) so t ther m.;si arel =	funct -3)(7 (x-2) here e are sturu 3, f	(+1) is a z-i c are has a	r de hole v.A	at cepts s at ot y=	xinz x=-: (3,(x=0	the 2 0), (c
Using Jormula :Xamples.	z-intera of a roo graph Holes z-ints V:A.s Since Since	epts, asympt tional function $f(x) = \frac{x^3}{x^3}$ first factor (x+2) is a (x-3),(x+1) and (x) and (if f is undefine the denomin on mous plot	otes, an from a f-5x-6 -4x the num factor of factors of x-2) are d for x ator has the asy	d test p graph. graph. of both th the nume factors c =0 (U.A.) 3 degree 3 motoles a	oints, od der e nu crator, of the there o, and	we a nomina marate but n oleno is r	ector: sr of der st the c m. but re y-ir numero s Th	ph a f(x) = rominoz lenom rot th tercept tor hose e = x - ix	(x- x) tor, so t nu s du	nal (2)(x (x+2) so t ther m.;si gree 5	funct -3)(7 (z-2) here e are s there 3, f	(+1) is a x-1 care has a s wh	r de hole v.a. HA. a	at cepts s at secti	x=-2 (3,0 X=0	the 2 0), (and (th

only effect end behavior, so when 0 < x < 2, it may cross the the However, the line may not cross the x axis, as there are no z-ints between 0 and 2. So a single test point will tell us what the line looks like. We check for x=1: $f(1) = \frac{1^3 - 611-6}{1^3 - 611-6} = \frac{1-6}{-3} = \frac{10}{3} > 0$. So the graph stays above

the x-axis between 0 and 2. Finally, we place the hole at x=-;

	Write a possible formula for the graph of fix
	Since there is a hole at 2-2, (2+2) must be a factor
	of both the numerator and denominator.
	Since there is an x-int at x=1, (x-1) must be a factor
	of the numerator
	Since there is a V.A. at x=-1, (x+1) must be a factor
	of the denominator
	Since the H.H. is at $y=-1$, the leading coefficient must be -1 $T1$, f_{1} , $-(y+z)(x-1)$
	We can check that this formula agrees with the y-intercept: $f(0) = \frac{-(0+2)(0-1)}{(0+2)(0+1)} = \frac{-(-2)}{2} = \frac{2}{2} = 1$
Domain	B
y fi	s a partial function on \mathbb{R} (e.g., a rational function or square root function) the domain of f is
the set c	fall real numbers & for which fex) is defined.
ig fous in word	= $\frac{p(x)}{q(x)}$, i.e., f is a rational function, then the domain of f is $\mathbb{R} \setminus \{x \in \mathbb{R} \mid q(x)=0\} = \{x \in \mathbb{R} \mid q(x)\neq 0\}$ s: the domain of f is all real numbers except for the zeros of the denominator.
y fa	= $\sqrt{g(x)} + $, i.e., f is a square root function, then the domain of f is $\{x \in \mathbb{R} \mid g(x) \ge 0\}$, i.e. all real numbers such that the involution is the domain of f is $\{x \in \mathbb{R} \mid g(x) \ge 0\}$, i.e. all real numbers such as a square root function.
examples.	$\frac{3x^{a}+2x-2}{(x-1)(x+3)} \text{has domain} \mathbb{R} \setminus \{1, -3\} = \{x \in \mathbb{R} \mid x \neq 1, -3\} = (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$
	$\int 2x-1+5$ has domain $\left\{x \in \mathbb{R} \mid x \ge \frac{1}{2}\right\} = \left[\frac{1}{2}, \infty\right]$, because $2x-1 \ge 0 \Leftrightarrow x \ge \frac{1}{2}$
Compositi	λλς
Compositi Y f; g a with the d	ons: ne partial functions on IR, the domain of fog is the domain of (the simplified form of) fog intersected iomain of g.
Compositi if f,g a with the d examples.	ons: ne partial functions on IR, the domain of fog is the domain of (the simplified form of) fog intersected comain of g. $f(x) = \frac{1}{x}$, $g(x) = \frac{x}{x+2}$. dom $f = \mathbb{R}\setminus\{0\}$, down $g = \mathbb{R}\setminus\{2\}$
Compositi if f;g a with the c examples.	ons: ne partial functions on IR, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2}. \text{dom } f = \mathbb{R} \setminus \{0\}, \text{ dom } g = \mathbb{R} \setminus \{2\}$ $f_{\text{og}}(x) = \frac{1}{x+2} = \frac{x+2}{x} \text{odom } f_{\text{og}} = \mathbb{R} \setminus \{0\}, \mathbb{R} \setminus \{-2\}$
Compositi if f;g a with the c examples.	ons: ne partial functions on R, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2} dom f = R \setminus \{0\}, dom g = R \setminus \{2\}$ $fog(x) = \frac{1}{\frac{x}{x+2}} = \frac{x+2}{x} dom fog = R \setminus \{0\}, R \setminus \{0\}, R \setminus \{0\}, -\frac{1}{x}\}$ $g \cdot f(x) = \frac{1}{\frac{1}{x+2}} = \frac{1}{1+2x} dom g \cdot f = R \setminus \{0\}, R \setminus \{0\}, -\frac{1}{x}\}$
Compositi if f;g a with the c examples.	ons: The partial functions on R, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2} \text{dom } f = R \setminus \{0\}, \text{ dom } g = R \setminus \{2\}$ $f_{og}(x) = \frac{1}{x+2} = \frac{x+2}{x} \text{odom } f_{og} = R \setminus \{0\}, \text{ dom } g = R \setminus \{2\}, \text{ dom } g = R \setminus \{2\}, \text{ dom } g = R \setminus \{2\}, \text{ dom } g = g = R \setminus \{2\}, \text{ dom } g = g = R \setminus \{2\}, \text{ dom } g = g = R \setminus \{2\}, \text{ dom } g = g = R \setminus \{2\}, \text{ dom } g = g = R \setminus \{2\}, \text{ dom } g = g = g = R \setminus \{2\}, \text{ dom } g = g = g = g = g = g = g = g = g = g$
Compositi if f,g a with the c examples.	ons: The partial functions on R, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2} \text{dom } f = R \setminus \{0\}, \text{ dom } g = R \setminus \{2\}$ $f_{0}(x) = \frac{1}{x+2} = \frac{1}{x} \text{odom } f_{0} = R \setminus \{0\}, R \setminus \{1\}, R \setminus \{1\},$
Compositi if f;g a with the c examples.	ons: The partial functions on R, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2} dom f = R \setminus \{0\}, dom g = R \setminus \{2\}$ $f_{og}(x) = \frac{1}{x+2} = \frac{x+2}{x} dom f_{og} = R \setminus \{0\}, dom g = R \setminus \{2\}$ $f_{og}(x) = \frac{1}{x+2} = \frac{x+2}{x} dom f_{og} = R \setminus \{0\}, dom g = R \setminus \{2\}$ $f_{og}(x) = \frac{1}{x+2} = \frac{1}{1+2x} dom f_{og} = R \setminus \{0\}, dom g = R \setminus \{2\}, dom g $
Compositi y f, g a with the c examples.	ons: ne partial functions on R, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2}. \text{dom } f = R \setminus \{0\}, \text{ dom } g = R \setminus \{2\}$ $f_{\sigma_{g}}(x) = \frac{1}{\frac{x}{x+2}} = \frac{x+2}{x} \text{odom } f_{\sigma_{g}} = R \setminus \{0\}, \text{ dom } g = R \setminus \{2\}$ $f_{\sigma_{g}}(x) = \frac{1}{\frac{x}{x+2}} = \frac{1}{1+2x} \text{odom } f_{\sigma_{g}} = R \setminus \{0\}, \text{ and } g$ $g \cdot f(x) = \frac{1}{\frac{x}{x+2}} = \frac{1}{1+2x} \text{odom } g \cdot f = R \setminus \{1\} \cap R \setminus \{0\}, \frac{1}{x}\}$ $h(x) = x^{2} + 3, k(x) = \sqrt{x^{-1}}. \text{dom } h = R, \text{dom } k - \{x \in R \mid x \ge 1\} = [1,\infty)$ $h_{\sigma}(x) = \sqrt{x^{2}+3} = x-1+3-x+2 \text{odom } h_{\sigma}k = R \cap \{x \in R \mid x \ge 1\} = \{x \in R \mid x \ge 1\}$ $h_{\sigma}(x) = \sqrt{x^{2}+3-1} = \sqrt{x^{2}+2}, \text{note that } x^{2}+2 = \infty x^{2}=-2, since a square in adways non-negative, this is true for all x \in R \\ n \in R \cap R = R$
Compositi if f, g a with the c examples.	ons: ne partial functions on IR, the domain of fog is the domain of (the simplified form of) fog intersected tomain of g. $f(x) = \frac{1}{x}, g(x) = \frac{x}{x+2}. dom f = R \setminus \{0\}, \ dom g = R \setminus \{-2\}$ $f \circ g(x) = \frac{1}{x+2} = \frac{x+2}{x} dom f \circ g = R \setminus \{0\}, \ dom g = R \setminus \{0, -2\}$ $g \circ f(x) = \frac{1}{x+2} = \frac{1}{1+2x} dom g \circ f = R \setminus \{1\}, \ R \setminus \{0\}, = R \setminus \{0\}, -2\}$ $g \circ f(x) = \frac{1}{x+2} = \frac{1}{1+2x} dom g \circ f = R \setminus \{1\}, \ R \setminus \{0\}, -\frac{1}{2}\}$ $h(x) = x^{2} + 3, \ k(x) = \sqrt{x-1}. dom h = R, \ dom k^{2} - \frac{1}{2} = R \setminus \{0, -\frac{1}{2}\}$ $h \circ k(x) = (\sqrt{x-1})^{2} + 3 = x-1+3 = x+2 on \ dom h^{2} = R \cap \{x \in R \mid x \geq 1\} = [1,\infty)$ $h \circ k(x) = \sqrt{x^{2}+3} = x-1+3 = x+2 on \ dom h^{2} = R \cap \{x \in R \mid x \geq 1\} = [x \in R \mid x \geq 1]$ $h \circ k(x) = \sqrt{x^{2}+3} = x-1+3 = x+2 on \ dom h^{2} = R \cap \{x \in R \mid x \geq 1\} = [x \in R \mid x \geq 1]$ $h \circ k(x) = \sqrt{x^{2}+3} = x-1+3 = x+2 on \ dom h^{2} = R \cap \{x \in R \mid x \geq 1\} = [x \in R \mid x \geq 1]$

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function	(or partial)	function)	is invertil	ble if a	nd only if it is	injective (one	-to-one	
Junction	is injective	if and o	 nly il pu	and output	t corresponds to	o unious inou	t. We can t	hink about this in tw
quivalent u	ays.						" . G Alvent	
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ormatly:	t is inject	ive yo	ma only i	Y Va,	bedomt, t(a	()= 1 (0) => u-0		
rraphically F passes	we say t the horizont	hat f is al line	injective test who	y and on un any f	nly if the graf cossible horizon	sh of f passes stal line on the	the horizontal c plane meet	line test. s the graph of f in
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camples.	.			passes	, so is injective	, so is invertible	L	
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	x ² ~	•		10.115 3	o is not injectiv			
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The do	main of	f" is	equal	to the	range of	f, fyerl	Jx.fc=y},ie	all the possible for
The do	main of	f ⁻¹ is	equal	to the	range of	f, fyerl	Jx.fc=y},i.e	, all the possible outputs of f(x)
The do To find	main of f ⁻¹ , swap c	f ⁻¹ is	equal ues of x	to the	range of and swap fi	f, fyerr x) with z, then	∃x.fæ=y},i.e . solue for y.	, all the possible outputs of $f(x)$
The do To find	main of f ⁻¹ , swap a f(x) =	f^{-1} is all instance $\frac{1}{\chi+3}$	equal ues of x ~ x=	to the with y $\frac{1}{y+3}$	range of and swap for $y+3 = \frac{1}{x}$	$f_{y=1}^{2} = \frac{1}{2} - 3$	$\exists x. f(x) = y^2$, i.e solue for y $\widehat{f^+}(x) = \frac{1}{x}$	outputs of f(x)
The do To find example.	main of f^{-1} , swap of f(x) = The range of	f ⁻¹ is ull instand x+3 f is RNS	equal ues of x ~ x=	to the with y $= \frac{1}{y+3} \sim$	and swap for $y+3 = \frac{1}{x}$ -	$f_{x} = \begin{cases} y \in \mathbb{R} \\ y \in \mathbb{R} \\ y = \frac{1}{x} - 3 \end{cases}$	f_x , $f_x = y_{1,i,e}^2$, $f_x = y_{1,i,e}^2$, $f_x = \frac{1}{x}$	outputs of f(x)
The do To find example.	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ull instand 1 x+3 6 f is RNS	equal ues of x ~ x= {o} (н.п. ы	to the with $y = \frac{1}{y+3}$	range of and swap for $y+3 = \frac{1}{x}$ tom f ⁻¹ = R(so)	$f_{y} = \frac{f_{y}}{x} - 3$	$\frac{3}{2} + \frac{1}{2} + \frac{1}$	outputs of f(x)
The do To find example.	main of f ⁻¹ , swop o f(x) = The range o	f^{-1} is $\frac{1}{x+3}$ f is RNS	205 of x ~ x = 203 (н.п. нт	to the with y = $\frac{1}{y+3}$ ~	range of and swap fit $y+3 = \frac{1}{x}$ - lom f ⁻¹ = R(50)	f, f yer () (x) with z , then (x) $y = \frac{1}{x} - 3$	$\exists x : f(x) = y i$, i.e. solue for y $\longrightarrow f^{-1}(x) = \frac{1}{x}$	-3
The do To find example.	main of f ⁻¹ , swap o f(x) = The range o	f^{-1} is $\frac{1}{x+3}$ f is R\S	equal es of x x = Eo3 (H.R. or	to the with y $=\frac{1}{y+3}$ $(y=0)$, so a	range of and swap fit $y + 3 = \frac{1}{x}$ - lom f ⁻¹ = R\(s)	$f = \int y e R $ (x) with z , then $y = \frac{1}{x} - 3$	$\exists x : f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	outputs of f(x)
The do To find example.	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ull instand x+3 f in RNA	equal ues of x ~ x = EQ (H.R. or	to the with y $=\frac{1}{y+3}$ \sim	range of and swap fi $y+3 = \frac{1}{x}$ - tom f ⁻¹ = R(50)	f, f yer R (3) with z , then (3) $y = \frac{1}{2} - 3$	$\exists x \cdot f(x) = y i$, i.e solue for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do To find example	main of f^{-1} , swap of f(x) = The range of	f ⁻¹ is ull instance x+3 6 f is RNS	сериаl 	to the with $y = \frac{1}{y+3}$ (rol), so a	range of and swap fit $y + 3 = \frac{1}{2}$ - lom f ⁻¹ = R(50)	f, f yer () (x) with x , then (x) $y = \frac{1}{x} - 3$	$\exists x : f(x) = y i$, i.e. solve for y \frown $f^{-1}(x) = \frac{1}{x}$	outputs of f(x)
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ull instance 1 x+3 6 f is RNS	6 equal 	to the with y y+3 ~ y=0); so a	range of and swap fit $y + 3 = \frac{1}{x}$ - lom f ⁻¹ = R(s)	$f = \int y e R $ (x) with z , then $y = \frac{1}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant 1 x+3 6 f in RNS	68 equal 108 of x 108 (H.A. M	to the with y 1 +3 y +0), so a	range of and swap f $\Rightarrow y+3 = \frac{1}{x}$ tom $f^{-1} = R \setminus \{0\}$	$f = \frac{1}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f^{-1} , swap of f(x) = The range of	f ⁻¹ is ull instant (x+3 (f is RN	equal ~ x = [0] (H.A. 41	to the with $y = \frac{1}{y+3}$ (1)	range of and swap fit $y + 3 = \frac{1}{2}$ - lom f ⁻¹ = R(so)	$f = \frac{1}{2} \left\{ y \in \mathbb{R} \right\}$	$\exists x \cdot f(x) = y i$, i.e. solue for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant x+3 6 f is RNS	6 equal 1.es of ∞ 1.es of ∞ 1.es of ∞	to the with y y+3 ~ y=0); so a	range of and swap fit $3 + 3 = \frac{1}{2}$ tom f ⁻¹ = R(s)	$f = \int y e R $ (x) with z , then $y = \frac{1}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant 1 x+3 6 f in RNS	6 equal 125 of X 103 (H.R. M	to the with y 1 1 1 1 1 1 1 1	tange of and swap fi $y+3 = \frac{1}{x}$ lon f ⁻¹ = R\(s)	$f = \frac{f}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f^{-1} is all instant	ies of x ios (11.6 or ios (11.6 or io) (11.6 or) (1	to the 	tange of and swap f $y+3 = \frac{1}{x}$ tom $f^{-1} = R \setminus \{0\}$	$f = \int y e R $	$\exists x \cdot f(x) = y i$, i.e. solue for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant (x+3 (f is RN	6 equal 1.25 of ∞ ~ X = [0] (H.R. of	to the with y y+3 ~ y=0) ; so a	range of and swap $f($ $\Rightarrow y+3 = \frac{1}{2}$ - lom $f' = R(s)$	$f = \int y e R $ (x) with z , then $y = \frac{1}{z} - 3$	$\exists x \cdot f(x) = y i$, i.e solue for y $\frown f^{-1}(x) = \frac{1}{x}$	-3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant x+3 6 f is RN	603 (H.A. 47	to the with y y+3 ~ y=0); so a	range of and swap find $y + 3 = \frac{1}{x}$ - loon f ⁻¹ = R(s)	$f = \int y e R $ (x) with z , then $y = \frac{1}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant 1 x+3 f in RN	60 (H.A. M	to the 	tange of and swap f : $y+3 = \frac{1}{x}$ tori $f' = R \setminus \{0\}$	$f = \frac{y}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solve for y $\frown f^{-1}(x) = \frac{1}{x}$	3
The do	main of f ⁻¹ , swap o f(x) = The range o	f^{-1} is all instant $\frac{1}{x+3}$ 6 f in RN	ies of x ios (H.A. or ios (H	to the 	tange of and swap f : $y+3 = \frac{1}{x}$ - tor $f^{-1} = R \setminus \{0\}$	f, f yer (f) (3) with z , then (3) $y = \frac{1}{2} - 3$	$\exists x \cdot f(x) = y i$, i.e. solue for y $\frown f^{-1}(x) = \frac{1}{x}$	3
The do	main of f ⁻¹ , swap o f(x) = The range o	f ⁻¹ is ult instant f is RNS f is RNS	ies of x ies of x ies (H.A. 41	to the 	tange of and swap f $y+3 = \frac{1}{2}$ - tom $f^{-1} = R \setminus \{0\}$	$f = \int y e R $	$\exists x \cdot f(x) = y i$, i.e. solue for y $\frown f^{-1}(x) = \frac{1}{x}$	3
The do	main of f ⁻¹ , swap c f(x) = The range o	f ⁻¹ is ult instant x+3 (f in RN	60 (H.A. 47	to the 	range of and swap find $y + 3 = \frac{1}{x}$ - loon f ⁻¹ = R.\so)	$f = \int y e R $ (x) with z , then $y = \frac{1}{x} - 3$	$\exists x \cdot f(x) = y i$, i.e. solue for y $\frown f^{-1}(x) = \frac{1}{x}$	3